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We call the sine and cosine transcendental functions and one might assume that their values as given in the usual table were, for the most part, transcendental numbers. So far from that being the case, none of the values of the natural functions listed in the usual trigonometric table are transcendental.

The proof is immediate. The angles whose functions are listed are all commensurable with π, being given in degrees and fractions of degrees. But if $\theta = m\pi/n$, where m and n are integers, then $\sin n\theta = \sin m\pi = 0$. But there is a well-known formula giving $\sin n\theta$ as a rational algebraic function of $\sin \theta$, if n is odd, or of $\sin \theta$ and $\cos \theta$, if n be even. In the latter case the equation may be rationalized in $\sin \theta$, and in either case on substituting zero for $\sin n\theta$ we find that $\sin \theta$ satisfies a rational algebraic equation with rational coefficients and is consequently an algebraic number. The fact that the values of the other trigonometric functions of θ are algebraic follows from the formulas connecting them with the value of $\sin \theta$.[1]

V. SLOPE OF A CURVE IN POLAR COÖRDINATES AT THE POLE.

By H. J. ETTLINGER, University of Texas.

The pole is an exceptional point in polar coördinates. For example to every point in the plane, except the pole, there corresponds one pair of coördinates, (ρ, θ), $\rho > 0$ and $0 \leqq \theta < 2\pi$. Conversely to each pair of such numbers there corresponds one point in the plane. The pole, however, corresponds to the single coördinate $\rho = 0$ and conversely.

At the pole, this fact produces a singular situation with respect to the slope of a curve, $\rho = f(\theta)$ or $\theta = F(\rho)$, which passes through it. At an ordinary point, the slope of the tangent is given as $m = \tan \varphi$, where $\varphi = \theta + \psi$ and ψ is the angle formed by the tangent at P with the radius OP. By the usual method, we find

$$\tan \psi = \rho \frac{d\theta}{d\rho} \cdot \tag{1}$$

At the pole this equation yields $\psi = 0$ and hence

$$\varphi = \theta. \tag{2}$$

To give a meaning to θ at $\rho = 0$, we consider a point P on the curve whose coördinates are $(\Delta\rho, \bar{\theta})$. As $\Delta\rho$ approaches zero, if $\bar{\theta}$ approaches a limit θ_0, this is the direction of the tangent to the curve at the pole. This justifies equation (2). To find this value of θ_0, we solve $f(\theta) = 0$ or $\theta_0 = F(0)$, where $F(\rho)$ is supposed continuous for $\rho = 0$. If the equation of the curve is so defined that to each value of ρ there corresponds only one value of θ, the slope of the tangent at the pole is uniquely determined as

$$m = \tan \theta_0.$$

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[1] On the other hand, any angle whose radian measure is a rational number does have transcendental numbers for the values of its trigonometric functions. See F. Klein, *Famous Problems of Elementary Geometry*, translation by Beman and Smith, Boston, 1897, p. 77.